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New Approach to Halo Orbit Determination and Control

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A simple, iterative numerical method for the determination of halo orbits of the circular restricted three-body problem is developed. The nonlinearities inherent to the halo orbit problem are treated as trajectory-dependent, persistent disturbance inputs. The method then utilizes a disturbance accommodating, linear state-feedback controller for the computation of a trajectory about a libration point that can be used as a fuel-efficient nominal path. It is also shown that the method can be used as an iterative method for generating a large, complex, quasiperiodic Lissajous trajectory starting with a first-order reference trajectory.

I. Introduction

IN the restricted three-body problem, originally formulated by Euler in 1772, two massive bodies, called primaries, exhibit two-body motion about their center of mass, called the barycenter. A third body with negligible mass is introduced into this system and does not affect the motion of the two primaries; its motion is determined by the gravitational field of the primaries. The equilibrium points, called libration or Lagrangian points, are points where the gravitational and centrifugal forces acting on the third body cancel each other. Euler had shown the existence of three collinear libration points in 1765 and Lagrange discovered the two triangular libration points in 1772.

The existence of periodic halo orbits around such libration points has been known for many years and has been the focus of much research in celestial mechanics. In the late 1960s and early 1970s, the use of libration points in the Earth–moon–spacecraft system was extensively studied by Farquhar[1,2] and Farquhar and Kamel.[3] Because the same face of the moon always faces the Earth, communications with the far side of the moon is impossible without a relay network. One method to provide this communications network, introduced by Farquhar, would be to position one communications satellite in a halo orbit about the translunar L_2 point. In addition, if another communications satellite is located at the cislunar L_1 point, there could be continuous communications coverage between the Earth and most of the lunar surface.

Halo orbits have also received attention for actual space missions. In November 1978 a spacecraft called the International Sun–Earth Explorer-3 was placed into a halo orbit around the interior sun–Earth L_1 point.[4,5] It remained in this orbit until June 1982. One of its mission objectives was to continuously monitor the characteristics of the solar wind and other solar induced phenomena, such as solar flares, about an hour before they could disturb the space environment near the Earth.

Much of the research dealing with halo orbits centered on describing the reference orbital trajectory accurately. This is of particular importance in reducing the fuel expenditure for a spacecraft forced to follow the reference orbital path.[1–6] The reference orbits often take the form of a series solution to the nonlinear equations of motion.

In this paper, a new approach to the orbit determination and control problem of a spacecraft in a halo orbit is presented. If a spacecraft is forced to follow a nominal path derived with the linearized equations of motion, it is subject to the dynamical nonlinearities inherent to the problem. Such nonlinearities are treated in this paper as the trajectory-dependent, persistent disturbance inputs because they are functions of the position, velocity, and acceleration of the spacecraft. A disturbance accommodating control technique, which has been successfully applied to a space station control problem[7,8] as well as flexible spacecraft control problems,[9,10] was also successfully utilized by Hoffman[11] to reduce the fuel expenditure for the halo orbit control problem in the Earth–moon–spacecraft system. The trajectory-dependent nature of the persistent disturbances, however, has not been considered by Hoffman.[11]

In this paper, the concept of disturbance accommodation is applied in an iterative manner to allow the spacecraft to deviate from the initial low-order reference trajectory and follow a trajectory that is closer to a quasiperiodic or periodic solution of the nonlinear equations of motion. The final trajectory, after several iterative disturbance accommodations, becomes very similar to a high-order reference trajectory that otherwise must be obtained analytically or numerically as a series solution to the nonlinear equations of motion.

II. Circular Restricted Three-Body Problem

This section briefly introduces the restricted three-body problem for the formulation of the halo orbit determination and control problem.

Nonlinear Equations of Motion

The restricted three-body problem is illustrated in Fig. 1. It is assumed that the mass m of the third body is insignificant compared to the masses M_1 and M_2 of the two primary bodies. Hence, the orbital motion of the two primary bodies is not affected by the third mass; that is, the motion is simply described by the two-body problem in which the two primary bodies rotate about their barycenter. It is further assumed that the two primary bodies rotate about their barycenter in circular orbits with constant angular velocity n. That is, we consider here the circular restricted three-body problem.

The position vector $\vec{R}$ of the third body relative to the barycenter is expressed in terms of basis vectors $\{\vec{i}, \vec{j}, \vec{k}\}$ of a rotating reference frame with an angular velocity vector of $n\vec{k}$ and with its origin at the barycenter as follows:

$$\vec{R} = X\vec{i} + Y\vec{j} + Z\vec{k} \tag{1}$$

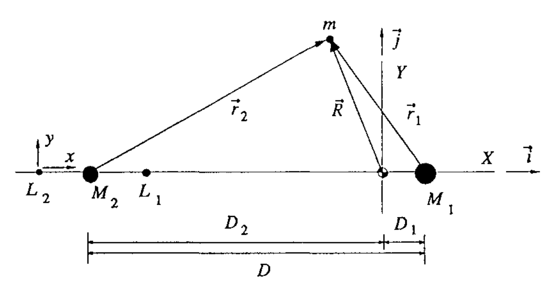

Fig. 1 Restricted three-body problem.

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In the standard formulation of the circular restricted three-body problem, the equations of motion are written in terms of a pseudopotential $U = U(X, Y, Z)$ as follows:

$$\ddot{X} - 2n\dot{Y} = \frac{\partial U}{\partial X} \tag{2a}$$

$$\ddot{Y} + 2n\dot{X} = \frac{\partial U}{\partial Y} \tag{2b}$$

$$\ddot{Z} = \frac{\partial U}{\partial Z} \tag{2c}$$

The pseudopotential U, which is the centrifugal plus gravitational force potential, is defined as

$$U = \tfrac{1}{2}n^2(X^2 + Y^2) + (\mu_1/r_1) + (\mu_2/r_2) \tag{3}$$

where $\mu_1 = GM_1$, $\mu_2 = GM_2$, G is the universal gravitation constant, and $r_1 = |\vec{r}_1|$ and $r_2 = |\vec{r}_2|$.

Introducing the mass ratio ρ of the three-body system as

$$\rho = M_2/(M_1 + M_2)$$

we rewrite the pseudopotential U in nondimensional form as

$$U = \tfrac{1}{2}(X^2 + Y^2) + [(1 - \rho)/r_1] + (\rho/r_2) \tag{4}$$

We then obtain the nonlinear equations of motion in nondimensional form as

$$\ddot{X} - 2\dot{Y} - X = -\frac{(1-\rho)(X-\rho)}{r_1^3} - \frac{\rho(X+1-\rho)}{r_2^3} \tag{5a}$$

$$\ddot{Y} + 2\dot{X} - Y = -\frac{(1-\rho)Y}{r_1^3} - \frac{\rho Y}{r_2^3} \tag{5b}$$

$$\ddot{Z} = -\frac{(1-\rho)Z}{r_1^3} - \frac{\rho Z}{r_2^3} \tag{5c}$$

where

$$r_1 = \sqrt{(X-\rho)^2 + Y^2 + Z^2}; \qquad r_2 = \sqrt{(X+1-\rho)^2 + Y^2 + Z^2}$$

and time is in units of $1/n$ and X, Y, Z, r_1, and r_2 are in units of D, the distance between M_1 and M_2.

For the Earth–moon system, we have $\rho = 0.01215$, $D = 384{,}748$ km, $D_1 = 4674$ km, $D_2 = 380{,}073$ km, and $n = 2.661699 \times 10^{-6}$ rad/s.

Libration Points

The locations of five libration points can be found with the use of the pseudopotential and are the solutions of the following three equations:

$$\frac{\partial U}{\partial X} = \frac{\partial U}{\partial Y} = \frac{\partial U}{\partial Z} = 0 \tag{6}$$

All possible libration points lie in the orbital plane of the two primaries. Euler discovered three libration points, called the collinear libration points, and Lagrange found two other libration points, called the equilateral libration points. These five libration points are denoted by L_1–L_5.

Linearized Equations of Motion

The linearized equations of motion with respect to a libration point (X_0, Y_0, Z_0) can be obtained in nondimensional form as

$$\ddot{x} - 2\dot{y} = \frac{\partial^2 U}{\partial X^2}x + \frac{\partial^2 U}{\partial Y \partial X}y \tag{7a}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial^2 U}{\partial X \partial Y}x + \frac{\partial^2 U}{\partial Y^2}y \tag{7b}$$

$$\ddot{z} = \frac{\partial^2 U}{\partial Z^2}z \tag{7c}$$

where x, y, and z are the components of the position vector of the spacecraft with respect to a libration point such that

$$X = X_0 + x; \qquad Y = Y_0 + y; \qquad Z = Z_0 + z$$

and the partial derivatives are evaluated at the libration point (X_0, Y_0, Z_0). Note that the out-of-plane, z-axis motion is decoupled from the in-plane, x- and y-axes motion.

The linearized equations of motion can also be written in state-space form as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} \tag{8}$$

where $\boldsymbol{x} = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$ and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ U_{XX} & U_{XY} & 0 & 0 & 2 & 0 \\ U_{YX} & U_{YY} & 0 & -2 & 0 & 0 \\ 0 & 0 & U_{ZZ} & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

For the L_2 point of the Earth–moon system to be considered in this paper, we have

$$U_{XX} = 7.3809; \qquad U_{YY} = -2.1904; \qquad U_{ZZ} = -3.1904$$

and $U_{XY} = U_{YX} = 0$. For these values of the L_2 point, the out-of-plane motion is simple harmonic with a nondimensional frequency of $\omega_z = 1.78618$ (period = 15.3 days). The in-plane motion has an unstable divergent mode ($\lambda_{1,2} = \pm 2.15868$) as well as an oscillatory mode with a nondimensional frequency of $\omega_{xy} = 1.86265$ (period = 14.7 days). It is noted that the period of the moon's orbit is 27.3 days.

Lissajous Trajectory and Halo Orbit

The solution of the linearized equations of motion can be made to contain only sine and cosine functions with the proper choice of initial conditions. It is shown in Ref. 12 that the conditions are met by choosing

$$\dot{x}(0) = (\omega_{xy}/\Gamma)y(0); \qquad \dot{y}(0) = -\omega_{xy}\Gamma x(0) \tag{10}$$

where

$$\Gamma = \frac{\omega_{xy}^2 + U_{XX}}{2\omega_{xy}}$$

In addition, if we choose $x(0) = z(0) = 0$ and $\dot{z}(0) = -y(0)\omega_z$, the solution of the linearized equations of motion further reduces to the following form:

$$x(t) = [y(0)/\Gamma]\sin\omega_{xy}t \tag{11a}$$

$$y(t) = y(0)\cos\omega_{xy}t \tag{11b}$$

$$z(t) = -y(0)\sin\omega_z t \tag{11c}$$

The difference between the frequencies of the in-plane and out-of-plane oscillations results in a quasiperiodic Lissajous trajectory. Unless the frequency ratio is a rational number, the Lissajous trajectory does not close. For the case of rational frequency ratios, the trajectory becomes periodic and is called a halo orbit. For most cases, the solution of the linearized equations of motion is not periodic, and some active control effort is needed to achieve equal in-plane and out-of-plane frequencies. This is often called period or frequency control in the literature. The resulting periodic orbit due to frequency control will also be called a halo orbit in this paper, although the term halo orbit in celestial mechanics usually means a larger, periodic orbit that is a solution of the nonlinear differential equations of motion.

In practice, the spacecraft will not follow the Lissajous trajectory or halo orbit naturally since it will not be possible to place a spacecraft into an orbit with such ideal initial conditions. The Lissajous trajectory and halo orbit derived from the linearized equations of motion, however, can be used as reference trajectories in the halo orbit control problem. Consequently, such trajectories will be called the Lissajous reference trajectory and halo reference orbit, respectively, throughout this paper.

III. Halo Orbit Control Problem

Problem Formulation

Since the collinear libration points are unstable for any restricted three-body system, orbit control is needed for a spacecraft to remain in the vicinity of these points. A primary goal of orbit control is to maintain the orbital stability of the spacecraft in a neighborhood of the nominal orbital path that can be either a Lissajous reference trajectory or a halo reference orbit.

For the halo orbit control problem considered in this paper, it is desired to maintain a 3500-km halo orbit about the translunar L_2 point. A spacecraft following this trajectory will always lie within the 11.1-deg beamwidth of a fixed lunar-surface antenna even when the latitudinal and longitudinal oscillations of the moon are taken into account.[2] A 3500-km Lissajous trajectory is also to be studied.

It is assumed that all of the states are available for feedback control. It is also assumed that the control acceleration is continuous and able to act in each direction of the rotating reference frame. The stationkeeping control problem of practical concern, including the effects of tracking and propulsion errors, is not addressed in this paper. Other practical issues regarding the use of constant low-thrust jets also are not addressed. This paper is concerned mainly with the computation of a fuel-efficient trajectory about a libration point using a disturbance accommodating control approach.

Equations of Motion

The nonlinear equations of motion of the spacecraft with control inputs can be found by simply adding the control acceleration (u_x, u_y, u_z) into Eq. (5) as follows:

$$\ddot{X} - 2\dot{Y} - X = -\frac{(1-\rho)(X-\rho)}{r_1^3} - \frac{\rho(X+1-\rho)}{r_2^3} + u_x \quad (12a)$$

$$\ddot{Y} + 2\dot{X} - Y = -\frac{(1-\rho)Y}{r_1^3} - \frac{\rho Y}{r_2^3} + u_y \quad (12b)$$

$$\ddot{Z} = -\frac{(1-\rho)Z}{r_1^3} - \frac{\rho Z}{r_2^3} + u_z \quad (12c)$$

where

$$r_1 = \sqrt{(X-\rho)^2 + Y^2 + Z^2}; \qquad r_2 = \sqrt{(X+1-\rho)^2 + Y^2 + Z^2}$$

Similarly, the linearized equations of motion of the spacecraft with the control input can be found by simply adding the control acceleration (u_x, u_y, u_z) into Eq. (8) as follows:

$$\dot{x} = Ax + Bu \quad (13)$$

where $x = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$, $u = [u_x, u_y, u_z]^T$, A is given by Eq. (9), and

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Linear Control Logic

Given the linear model of the system described by Eq. (13), we can consider the state feedback control logic

$$u = -K(x - x_r) \quad (14)$$

where K is the constant gain matrix to be determined and

$$x_r = [x_r, y_r, z_r, \dot{x}_r, \dot{y}_r, \dot{z}_r]^T \quad (15)$$

is the reference orbit state vector described by

$$x_r(t) = [y(0)/\Gamma] \sin \omega_{xy} t \quad (16a)$$

$$y_r(t) = y(0) \cos \omega_{xy} t \quad (16b)$$

$$z_r(t) = -y(0) \sin \omega_z t \quad (16c)$$

The linear-quadratic-regulator (LQR) method can be simply employed to determine the gain matrix K by selecting proper weighting matrices Q and R of the following LQR performance index:

$$J = \frac{1}{2} \int_0^\infty (x^T Q x + u^T R u)\, dt \quad (17)$$

Nonlinear Simulation Results

A Macintosh personal computer with the software package MATLAB/Simulink was used to design a linear state-feedback controller and also to simulate the closed-loop nonlinear dynamical system.

After several trial-and-error design iterations, we select the following gain matrix in nondimensional form:

$$K = \begin{bmatrix} 39.016 & -7.154 & 0 & 9.876 & 0.122 & 0 \\ 7.329 & 28.690 & 0 & 0.122 & 8.823 & 0 \\ 0 & 0 & 28.593 & 0 & 0 & 8.786 \end{bmatrix}$$

Note that the decoupled nature of the in-plane and out-of-plane dynamics has resulted in a gain matrix with many zero elements.

The linear state-feedback controller was successful in maintaining the Lissajous reference trajectory, as well as the halo reference orbit, despite the nonlinear dynamical effects. The actual trajectories are quite similar to the reference trajectories. The control acceleration inputs needed to maintain such reference trajectories are shown in Figs. 2 and 3 for the Lissajous trajectory and halo orbit, respectively. The cyclic nature of the control acceleration inputs is a result

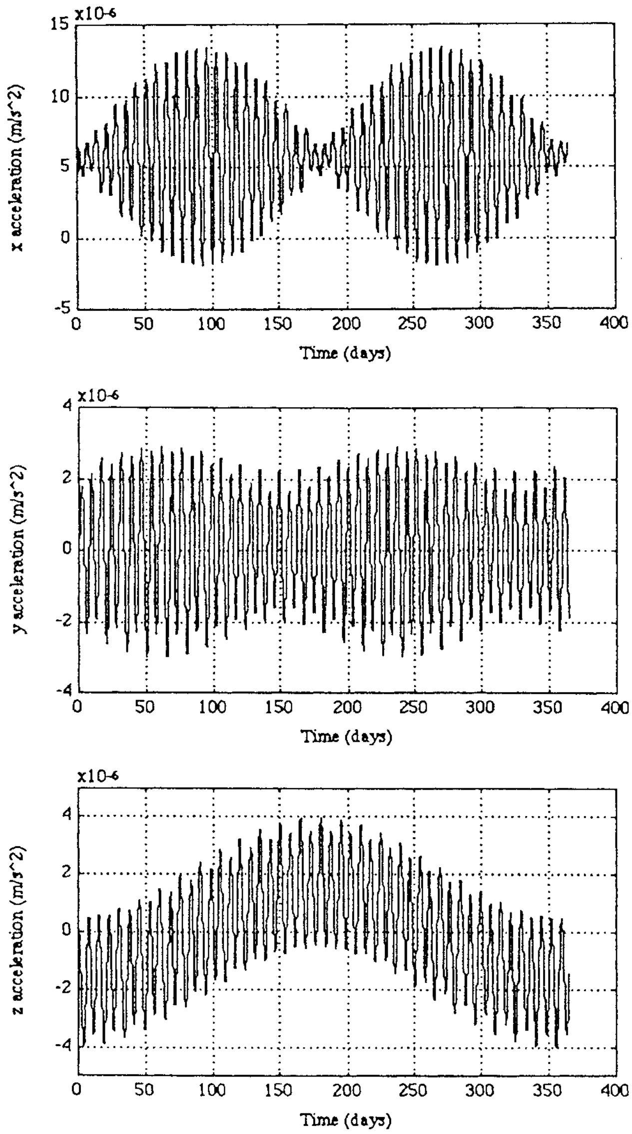

Fig. 2 Control inputs for tracking Lissajous reference trajectory.

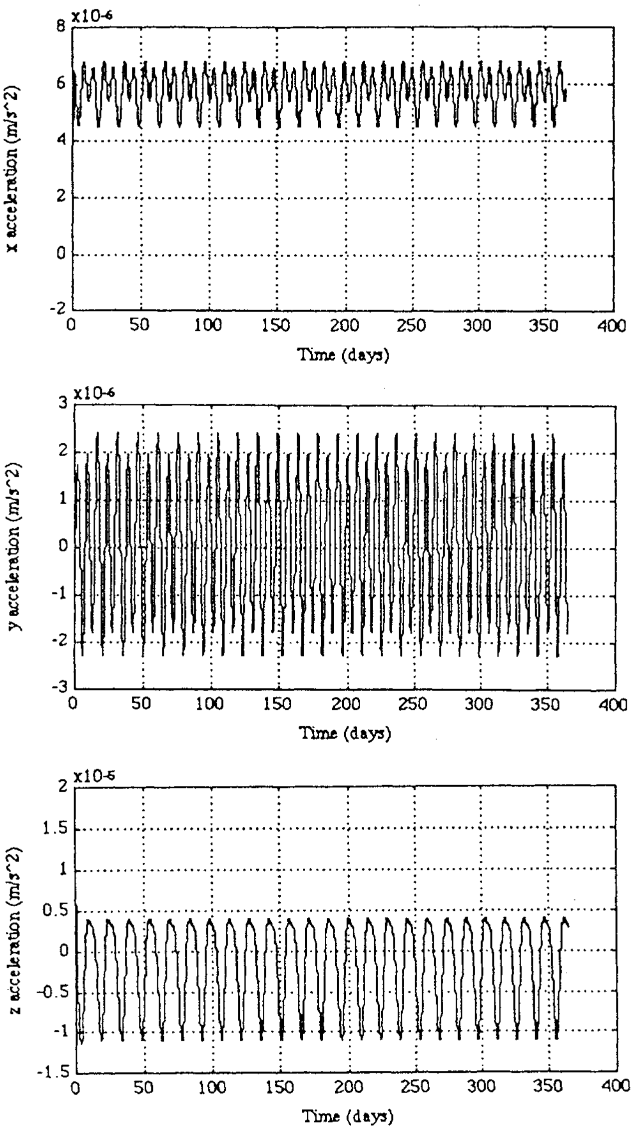

Fig. 3 Control inputs for tracking halo reference orbit.

of the nonlinear dynamical effects neglected in the derivation of the linear reference trajectories. The ΔV was found by integrating the control acceleration inputs, and for both the Lissajous trajectory and the halo orbit, the ΔV is very large in comparison to a geosynchronous satellite. The average ΔV for a geosynchronous satellite is about 50 mps/year. It was found that a spacecraft would require approximately 285 mps/year to maintain a linear Lissajous trajectory and approximately 375 mps/year to maintain a linear halo orbit. The additional ΔV for the halo reference orbit is attributed to the z-axis period control.

These results explain why there is interest in developing more accurate, nonlinear reference trajectories that are closer to an exact periodic solution of the nonlinear equations of motion. These more accurate reference trajectories provide a considerable reduction in ΔV at the expense of including many terms in the reference trajectory. It was shown in Refs. 1 and 2 that a ΔV of approximately 23.5 mps/year is needed when a third-order Lissajous reference trajectory is used and a ΔV of approximately 132 mps/year is needed when a more accurate (nonlinear) halo reference orbit is used. (These estimates in Refs. 1 and 2 were determined with a system model that included lunar orbital eccentricity and solar effects.)

IV. Disturbance Accommodating Control

In this section, the concept of disturbance accommodation, which was successfully applied in Refs. 7–11 for various linear control problems, is utilized to reduce the fuel expenditure (i.e., ΔV) by allowing the spacecraft to deviate from the commanded reference trajectory derived with the linearized equations of motion and follow a trajectory that is closer to a periodic solution of the nonlinear equations of motion.

The nonlinear dynamical effects are modeled as constant and cyclic disturbance inputs. It is again emphasized that these disturbance inputs are functions of the position, velocity, and acceleration of the spacecraft; they are, therefore, termed trajectory-dependent persistent disturbance inputs. For this reason, an iterative approach for the design of a disturbance accommodating controller is introduced, and the approach successfully allows the spacecraft to deviate from the reference trajectory and follow a trajectory that requires substantially less ΔV.

Characterization of Nonlinear Dynamical Effects

The frequency components of the control acceleration needed to maintain a reference trajectory are first determined for the design of a disturbance accommodating controller. The nonlinear equations of motion, Eqs. (12), are used to determine the control acceleration inputs as

$$u_x = \ddot{X} - 2\dot{Y} - X + \frac{(1-\rho)(X-\rho)}{r_1^3} + \frac{\rho(X+1-\rho)}{r_2^3} \quad (18a)$$

$$u_y = \ddot{Y} + 2\dot{X} - Y + \left[(1-\rho)Y/r_1^3\right] + \left[\rho Y/r_2^3\right] \quad (18b)$$

$$u_z = \ddot{Z} + \left[(1-\rho)Z/r_1^3\right] + \left[\rho Z/r_2^3\right] \quad (18c)$$

where

$$r_1 = \sqrt{(X-\rho)^2 + Y^2 + Z^2}; \qquad r_2 = \sqrt{(X+1-\rho)^2 + Y^2 + Z^2}$$

The orbital position of the spacecraft following a reference trajectory is described by

$$X = X_0 + x_r; \quad Y = Y_0 + y_r; \quad Z = Z_0 + z_r \quad (19)$$

Considering a collinear libration point with $Y_0 = Z_0 = 0$ and substituting Eqs. (19) into Eq. (18), we obtain

$$u_x = \ddot{x}_r - 2\dot{y}_r - (X_0 + x_r) + (1-\rho)(X_0 + x_r - \rho)r_1^{-3}$$
$$+ \rho(X_0 + x_r + 1 - \rho)r_2^{-3} \quad (20a)$$

$$u_y = \ddot{y}_r + 2\dot{x}_r - y_r + (1-\rho)y_r r_1^{-3} + \rho y_r r_2^{-3} \quad (20b)$$

$$u_z = \ddot{z}_r + (1-\rho)z_r r_1^{-3} + \rho z_r r_2^{-3} \quad (20c)$$

where

$$r_1^{-3} = \left[(X_0 + x_r - \rho)^2 + y_r^2 + z_r^2\right]^{-\frac{3}{2}}$$

$$r_2^{-3} = \left[(X_0 + x_r + 1 - \rho)^2 + y_r^2 + z_r^2\right]^{-\frac{3}{2}}$$

This is an analytical expression for the control acceleration inputs needed to force a spacecraft to follow a reference trajectory about one of the collinear libration points. If the reference trajectory (x_r, y_r, z_r) is, in fact, a periodic solution of the nonlinear equations of motion, then the control accelerations (u_x, u_y, u_z) will be equal to zero.

Substituting the linear reference trajectory, described by Eqs. (16), into Eqs. (20), we can estimate the frequency components of the control inputs needed to maintain the given reference trajectory. A discrete fast Fourier transform code of MATLAB™ was used to obtain the power spectral density (PSD) plots of u_x, u_y, and u_z. The spectral components of the resulting PSD plots represent the nonlinear dynamical effects neglected in the derivation of the linear reference trajectories. Similar spectral components can also be found in a third-order analytic solution by Farquhar[2] and Farquhar and Kamel.[3] Although the solution in Refs. 2 and 3 includes lunar orbital eccentricity and solar effects, the terms resulting from nonlinearities can be easily identified. Disturbance accommodation removes these components from the control acceleration inputs and allows them to show up in the orbital motion of the spacecraft.

Note that the analytic series solutions of Farquhar[2] and Farquhar and Kamel[3] contain bias, sine, and cosine terms with frequencies similar to those included in the disturbance accommodating

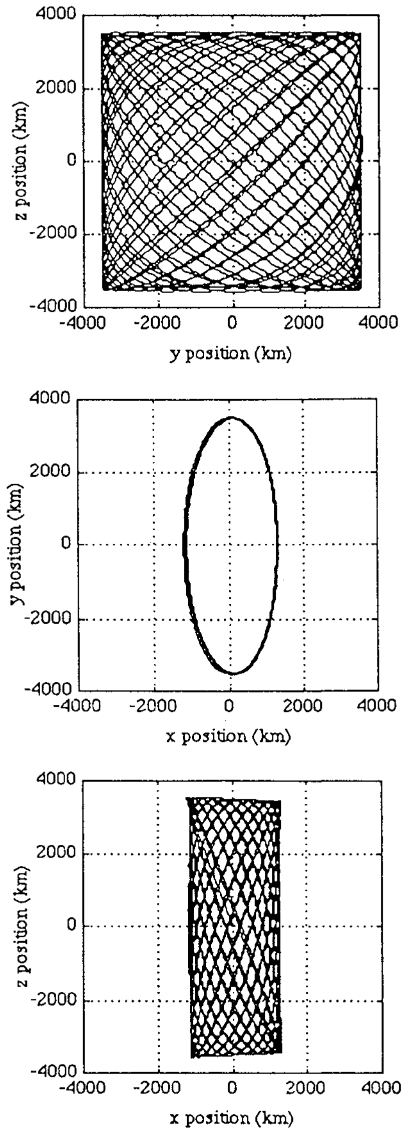

Fig. 4 Fuel-efficient Lissajous trajectory after disturbance accommodation.

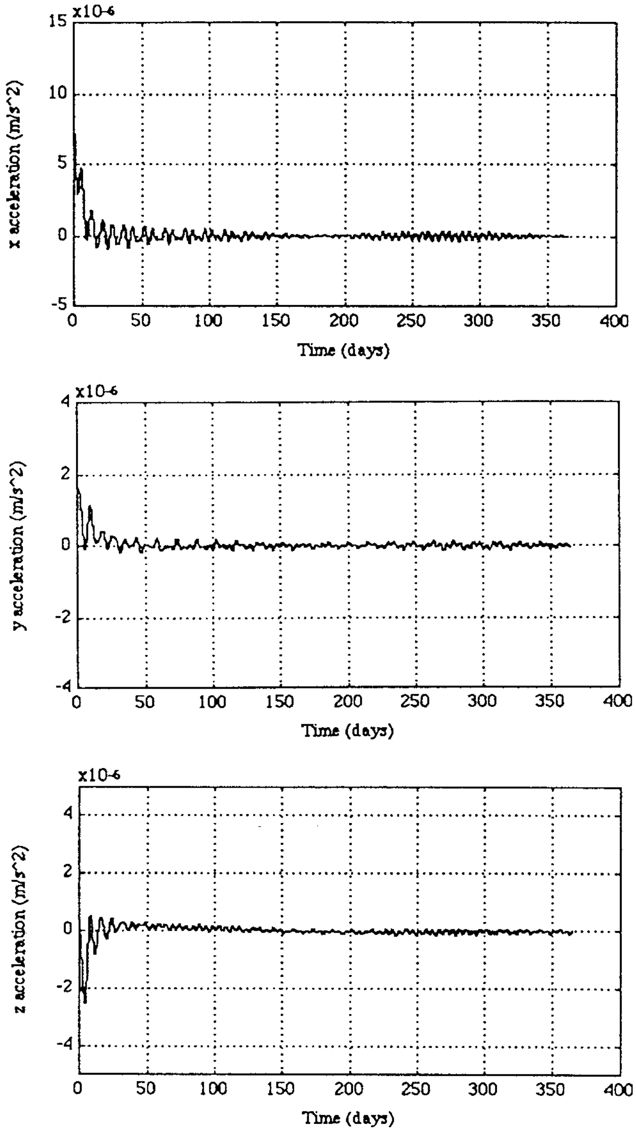

Fig. 5 Control inputs for fuel-efficient Lissajous trajectory after disturbance accommodation.

controller. This should be expected because disturbance accommodation allows the effects from nonlinearities to show up in the orbital motion of the spacecraft. Therefore, the iterative design of a disturbance accommodating controller and an analytic solution, in fact, produce similar results. A major advantage of disturbance accommodation, however, is that only the frequency of the disturbance needs to be determined. To include these frequencies in an analytic solution, the amplitude, frequency, and phase must also be determined accurately.

Disturbance Accommodating Control Design

The disturbance accommodation filter for the control inputs u_x, u_y, and u_z can be described by

$$\ddot{\alpha} + \omega_x^2 \alpha = u_x; \qquad \dot{\tau}_x = u_x$$
$$\ddot{\beta} + \omega_y^2 \beta = u_y; \qquad \dot{\tau}_y = u_y$$
$$\ddot{\gamma} + \omega_z^2 \gamma = u_z; \qquad \dot{\tau}_z = u_z$$

where τ_x, τ_y, and τ_z are the disturbance filter states necessary to eliminate any bias components of the control inputs; and ω_x, ω_y, and ω_z are the frequency components of the control inputs. The disturbance filter can include as many frequencies as the given persistent disturbance model and is driven by the control inputs. The disturbance accommodation filter is then described in state-space form as

$$\dot{\boldsymbol{x}}_d = \boldsymbol{A}_d \boldsymbol{x}_d + \boldsymbol{B}_d \boldsymbol{u} \tag{21}$$

where $\boldsymbol{x}_d$ is the disturbance filter state vector.

The disturbance filter described by Eq. (21) can then be augmented to the plant described by Eq. (13) to yield

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{x}}_d \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & 0 \\ 0 & \boldsymbol{A}_d \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x}_d \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{B}_d \end{bmatrix} \boldsymbol{u} \tag{22}$$

The standard LQR design technique can be similarly applied to the augmented system described by Eq. (22) for the design of a linear state-feedback controller of the form

$$\boldsymbol{u} = \boldsymbol{K} \begin{bmatrix} \boldsymbol{x}_r - \boldsymbol{x} \\ -\boldsymbol{x}_d \end{bmatrix}$$

A detailed discussion of the disturbance accommodating control design can be found in Ref. 13.

Nonlinear Simulation Results

Lissajous Reference Trajectory

Table 1 contains the frequencies that are included in the design of a disturbance accommodating controller for the case of a Lissajous reference trajectory. The spectral components of Eq. (20) were first used in the design of a disturbance accommodating controller. The results of a nonlinear simulation indicated that the control input u_y was still responding to nonlinear effects, and a PSD plot of the control input u_y indicated a frequency of $2\omega_z$. This frequency was then included in the redesign. The fact that this frequency component showed up in u_y can be attributed to the coupled x–y equations of motion and to the trajectory dependence of nonlinear effects.

Figure 4 shows the orbital motion of the spacecraft for one year with the linear Lissajous reference trajectory and the disturbance accommodating controller. The actual trajectory is very similar to the reference trajectory, but slight deviations are observed. Disturbance accommodation, however, gives a considerable reduction in the control acceleration, as shown in Fig. 5. The ΔV is estimated to be approximately 10 mps/year for this case, which is much less than the ΔV of 285 mps/year without disturbance accommodation.

Halo Reference Orbit

Table 2 contains the frequencies that are included in the design of a disturbance accommodating controller for the case of a halo reference orbit. Note that the linear halo reference orbit is obtained by simply setting the out-of-plane frequency (ω_z) equal to the in-plane frequency (ω_{xy}), which is often referred to as period control. Thus, it is not possible to reject all of the control frequency components for the halo orbit case. The z-axis control frequency associated with period control is ω_{xy}. If this component is rejected in u_z, then the spacecraft follows a Lissajous trajectory. Unfortunately, this is the largest component of the control acceleration. The z-axis period control results in the halo orbit but requires considerably more control acceleration than the case of a Lissajous trajectory even after disturbance accommodation.

Figure 6 shows the orbital motion of the spacecraft for one year with the halo reference orbit and the disturbance accommodating controller. Figure 7 shows the control acceleration required to maintain the halo orbit after disturbance accommodation. The z-axis period control can be easily seen in the u_z plot of Fig. 7. The ΔV is estimated to be approximately 140 mps/year. Note that the large ΔV required for this case is attributed mainly to the z-axis period control. It is still less, however, than the ΔV of 375 mps/year without disturbance accommodation.

Table 1 Spectral components of the control inputs for Lissajous reference trajectory

x axis	y axis	z axis
0	0	0
$2\omega_z$	$2\omega_{xy}$	$\omega_{xy} - \omega_z$
$2\omega_{xy}$	$2\omega_z$	$\omega_{xy} + \omega_z$

Table 2 Spectral components of the control inputs for halo reference orbit

x axis	y axis	z axis
0	0	0
ω_{xy}	$2\omega_{xy}$	ω_{xy}
$2\omega_{xy}$		

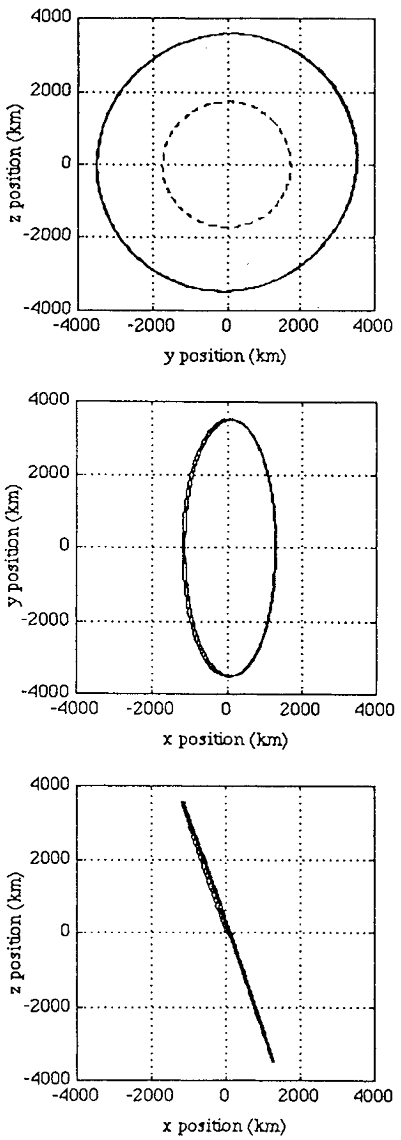

Fig. 6 Fuel-efficient halo orbit after disturbance accommodation.

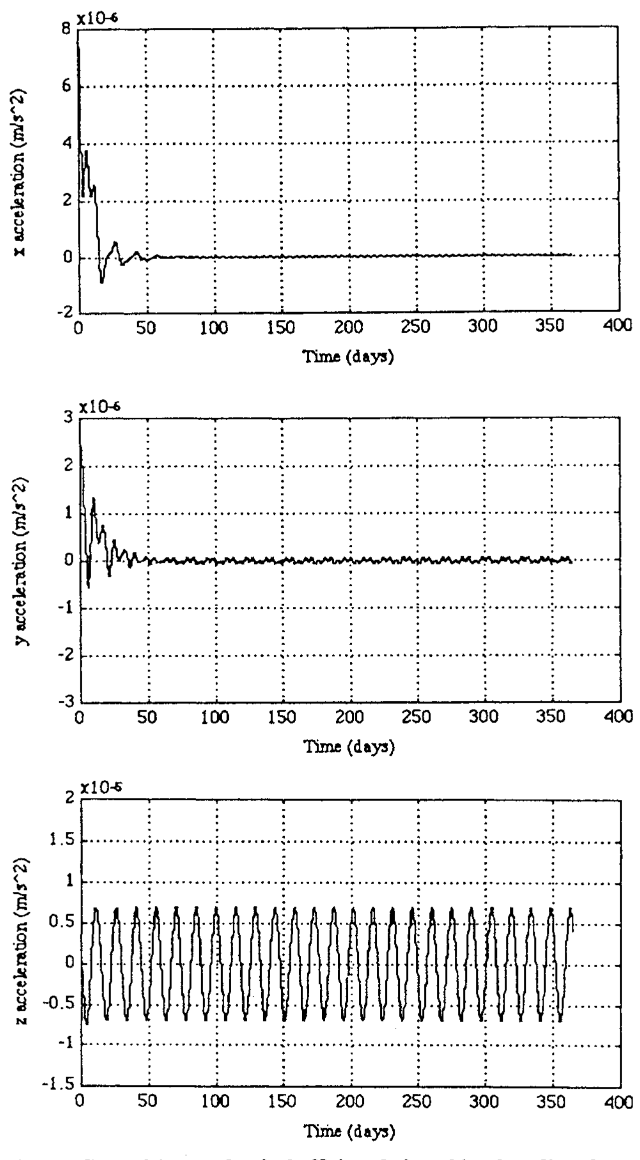

Fig. 7 Control inputs for fuel-efficient halo orbit after disturbance accommodation.

V. Large Lissajous Trajectory

As shown in the preceding section, the z-axis period control is needed to guarantee no periods of lunar occultation but requires a large ΔV. If this constraint can be relaxed and small periods of lunar occultation are allowed, a Lissajous trajectory may be used to provide lunar far-side communications. A large Lissajous trajectory with an amplitude of 30,000 km is considered here to compare the results of disturbance accommodation with the results of Howell and Pernicka.[6]

The large orbit size will result in the spacecraft being far away from the linear domain about the libration point. The solution of the linearized equations of motion, however, was still used as an initial reference orbit for the determination of a trajectory that is closer to a solution of the nonlinear equations of motion. As a result of the spacecraft being far away from the linear domain, there is a large number of frequencies required to accommodate the nonlinear effects. This case further demonstrates that the disturbances are, in fact, trajectory dependent. For example, when constant disturbance accommodation was included in the first design iteration the spectral components of the disturbances changed considerably for the next iteration step.

The results of an iterative application of disturbance accommodation are presented in Table 3. The trajectory with a controller found after 12 iterations is shown in Fig. 8. This somewhat peculiar trajectory in Fig. 8 is, however, very similar to the result in Ref. 6 that was obtained using a numerical method.

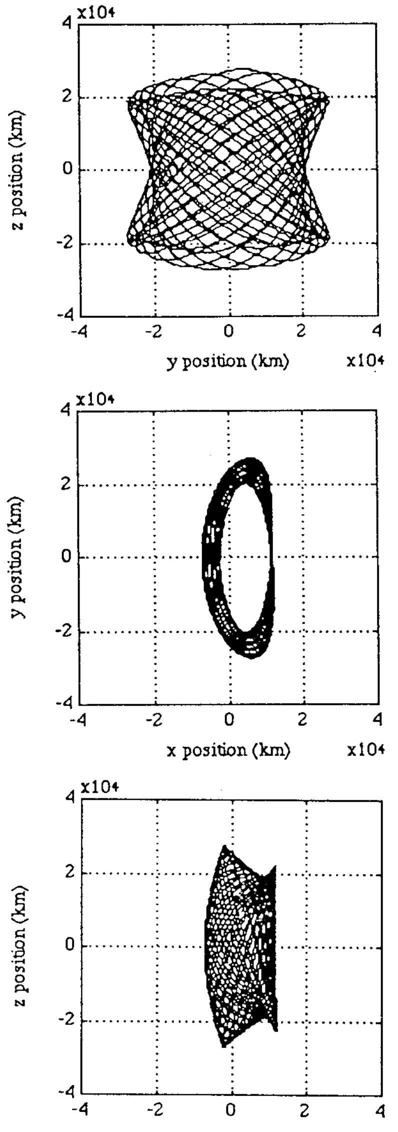

Fig. 8 Fuel-efficient, large Lissajous trajectory after 12 iterations.

Table 3 Design iterations for fuel-efficient large Lissajous trajectory

	x-axis frequencies	y-axis frequencies	z-axis frequencies	ΔV (mps/year)
0				18,517
1	0	0	0	14,652
2	$2\omega_z$ $2\omega_{xy}$			9,010
3	$2\omega_z + \omega_{xy}$ $3\omega_{xy}$	$2\omega_z$	$\omega_{xy} + \omega_z$	3,036
4	ω_{xy} $2\omega_{xy} + 2\omega_z$ $4\omega_{xy}$	$2\omega_z - \omega_{xy}$ $2\omega_z + \omega_{xy}$	$2\omega_{xy} - \omega_z$ $2\omega_{xy} + \omega_z$	2,233
5	$2\omega_{xy} - 2\omega_z$ $2\omega_z - \omega_{xy}$	$2\omega_{xy} - 2\omega_z$ $2\omega_{xy} + 2\omega_z$	$\omega_{xy} - \omega_z$ $3\omega_z + \omega_{xy}$	1,427
6	$4\omega_z + \omega_{xy}$ $2\omega_z + 3\omega_{xy}$			1,267
7	$4\omega_z - 2\omega_{xy}$ $4\omega_z - \omega_{xy}$ $4\omega_z$	$2\omega_{xy}$ $4\omega_z + \omega_{xy}$ $2\omega_z + 3\omega_{xy}$	ω_z $3\omega_z - \omega_{xy}$ $3\omega_{xy} - \omega_z$ $3\omega_z$	466
8		$3\omega_{xy}$		362
9	$2\omega_z + 4\omega_{xy}$	$4\omega_{xy} - 2\omega_z$	$3\omega_z + 2\omega_{xy}$	293
10	$3\omega_{xy} - 2\omega_z$	ω_{xy} $3\omega_{xy} - 2\omega_z$ $4\omega_{xy}$ $2\omega_z + 4\omega_{xy}$	$3\omega_z - 2\omega_{xy}$	162
11	$4\omega_{xy} - 2\omega_z$			152
12		$4\omega_z - \omega_{xy}$ $4\omega_z$		143

Further demonstration of the applicability of the proposed method can be found in Ref. 14 for the elliptic restricted three-body problem. In Ref. 14, the effects from both orbital eccentricity and dynamical nonlinearities are modeled as trajectory-dependent persistent disturbances, and the iterative method utilizes a disturbance accommodating linear state-feedback controller to determine fuel-efficient, quasiperiodic or periodic orbits. The method is applied to the elliptic restricted three-body problem of the Earth–moon–spacecraft system as well as the sun–Earth/moon–spacecraft system in Ref. 14.

VI. Conclusions

A simple, iterative numerical approach to the halo orbit determination and control problem has been presented. The nonlinearities inherent to the halo orbit control problem were modeled as constant and periodic trajectory-dependent disturbances to allow the application of disturbance accommodation. Disturbance accommodation was applied in an iterative manner to allow the spacecraft to deviate from the solution of the linearized equations and follow a trajectory that was closer to a quasiperiodic or periodic solution of the nonlinear equations of motion.

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